

1995-96 School Year

Mathematics 30 Information Bulletin

Diploma Examinations Program

This document was written primarily for:

Students	✓
Teachers	✓ Mathematics 30 teachers
Administrators	✓
Parents	
General Audience	
Others	

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Purpose of the Bulletin

The purpose of this bulletin is to provide students and teachers of Mathematics 30 with information about the diploma examinations scheduled for 1996.

We encourage teachers to share the contents of this bulletin with students and to review the scoring criteria with them.

If you have requests, questions, or comments about the contents of this bulletin, please contact:

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Important Dates in 1996

Administration of the Examination

<i>1996 Administrations</i>	<i>Time*</i>
Monday, January 29	9:00–11:30 A.M.
Wednesday, June 26	9:00–11:30 A.M.
Wednesday, August 14	9:00–11:30 A.M.

* The diploma examination is designed for a writing time of 2.5 h. Students may have an additional 0.5 h to complete the examination.

Scoring of the Examination

<i>1996 Administrations</i>	<i>Scoring Dates**</i>
January	February 6 and 7 (Head Markers) February 8 to 10
June	July 7 and 8 (Head Markers) July 9 to 12
August	August 16

** Dates are tentative and will be confirmed by telephone with markers.

Notes of Interest

Information About Markers

The written-response questions of the Mathematics 30 diploma examinations are marked by Mathematics 30 teachers selected from those who have been recommended as markers to the Student Evaluation Branch by their superintendents. To qualify for recommendation, a teacher must have taught the complete Mathematics 30 course twice, be teaching the course in the current school year, and have an Alberta or Northwest Territories Permanent Professional Teaching Certificate.

Often, more teachers are recommended as markers by superintendents than are required by the Student Evaluation Branch for any one marking session. The following criteria are considered when markers are selected for a particular marking session:

- experience as a marker (experienced markers and first-time markers are included)
- regional representation (by zone, jurisdiction, and school)
- student population (proportional representation)

Teachers are particularly needed who can mark examinations written in French.

Teachers who wish to be recommended as markers for the January 1996 examination should contact their superintendents by September 22, 1995. Teachers who wish to be recommended as markers for the June and August 1996 examinations should contact their superintendents by **February 23, 1996.**

Item Writing and Field Testing

Each school year, letters are sent to superintendents requesting their nominations for teachers to participate in field testing and item writing. **Teachers who are interested in these activities should let their superintendents know by September 22, 1995.**

Reports

After each January and June administration of the Mathematics 30 diploma examination, copies of the *Examiners' Report* for that particular administration are sent to teachers in all senior high schools in Alberta. The report contains provincial results along with comments about student performance on the examination.

Examiners' reports are designed for teacher use. If we can make these reports more useful to you, please let us know.

School and Jurisdiction Statistical Reports are sent to each superintendent and senior high school principal after each January and June administration of the Mathematics 30 diploma examination. These reports provide detailed information on how well students in their school district and school, respectively, did on the Mathematics 30 diploma examination. Teachers may use these data to reflect on the areas of the program where their students did meet standards and those areas where their students did not meet the standards.

Each year, copies of a document entitled *Annual Report, Diploma Examination Program*, are sent to each superintendent and senior high school principal in Alberta. The report contains information about the results achieved by students who wrote diploma examinations in the school year. It also contains special studies on topics of interest. The first annual report (1989–90 school year) contained a study comparing achievement in various diploma courses; the second (1990–91 school year) contained a study comparing the achievement of students who repeat diploma examination courses and rewrite diploma examinations with their achievement the first time and with the achievement of students who write only once; the third report (1991–92 school year) contained a study of participation rates in diploma examination courses; the fourth report (1992–93 school year) contained a study of English 30 students' application of conventions of language; and the fifth report (1993–94 school year) presented a comparison of provincial participation rates in diploma examination subjects for the school years 1991–92, 1992–93, and 1993–94. Each of these special studies will be of interest to teachers who are involved in interpreting the diploma examinations results of their students.

Inservices and Presentations

On a limited basis and subject to budget, Student Evaluation Branch staff is available to provide inservices or presentations related to diploma examinations or the interpretation of diploma examination results.

Student Use of Scientific Calculators

Examinations are constructed to ensure that the use of particular scientific calculators neither advantages nor disadvantages individual students.

Please refer to Appendix A for the policy statement on the use of scientific calculators on diploma examinations. Students should be made aware of this policy as early as possible in the school term to ensure they are able to use the scientific calculator of their choice when writing examinations.

Students should also be made aware of the Examination Rules, Grade 12 Diploma Examinations (see Appendix B). Students should know that notes stored in electronic devices may not be brought into the examination room.

Permitted

The Examination Rules also state: “Students may use calculator programs designed to perform mathematical computations and/or those designed to assist students in graphing.” An example of a program that would assist students with graphing is the program for graphing quadratic relations to be found in the *Mathematics 30/33 Interim Teacher Resource Manual*. A second example of a program that students could use is a program designed to determine values in the quadratic formula.

Data Resources for Students

There is a tear-out formula sheet and a z-score page provided in all 1996 examinations.

Student Requests for Rescores

Students may request a rescoring of their diploma examinations. Before applying for a rescoring, students should check their Diploma Examination Results Statement to find out the distribution of marks. The mark for the machine-scored questions is not likely to change as a result of a rescore, but the written-response mark could change slightly. Students should remember that the rescored mark will be the final mark whether it *increases* or *decreases*.

Students who decide to have an examination rescored must ensure that their application is received before the deadline specified on the results statement. The fee for rescoring each examination is \$26.75, which includes GST. If a diploma examination mark is increased by 5% or more as a result of rescoring, the fee is refunded.

Student Requests for Rewriting

Students may rewrite a diploma examination in any regularly scheduled administration. Students must apply to the Student Evaluation Branch by November 15, 1995, and April 15, 1996 to be eligible to write the January and June diploma examinations. The fee for rewriting each examination is \$26.75, which includes GST. (For more details, see the *General Information Bulletin*.)

Mathematics 30 Information

Standards

Provincial standards help to communicate how well students need to perform to be judged as having achieved the learnings specified for Mathematics 30. According to the *Mathematics 30 Course of Studies*, student learnings refer to specific knowledge, skill, and attitude expectations. These learnings are amplified in Appendix C, Mathematics 30 Curriculum Standards, of this bulletin. Included in Appendix C are examples of questions that students must be able to answer to demonstrate *acceptable* or *excellent* achievement.

Students who demonstrate *acceptable* achievement but not *excellent* achievement in Mathematics 30 will receive a final course mark between and including 50% and 79%. Typically, these students have gained new skills and knowledge in mathematics but can anticipate difficulties if they choose to enroll in post-secondary mathematics courses. They have demonstrated mathematical skills and knowledge in the seven content strands of the Mathematics 30 curriculum and exhibit an ability to apply a broad range of problem-solving skills to these content strands.

Students who demonstrate *excellent* achievement will receive a final course mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and feel confident about their mathematical abilities. These students should encounter little difficulty in post-secondary mathematics programs; they should be encouraged to pursue careers in which they will use their talents in mathematics.

The specific statements of standards that follow were written primarily to inform Mathematics 30 teachers about the extent to which students must know the Mathematics 30 content and must demonstrate the required skills to pass the examination. The examples provided are by no means exhaustive: they are intended to provide a profile of *acceptable* and *excellent* achievement.

Problem Solving

Students in Mathematics 30 should be able to participate in and contribute toward the problem-solving process for problems within the seven content strands.¹

The student demonstrating acceptable achievement can:

- Given the solution to a problem, analyze the solution for correctness, provide the correct response, and provide possible reasons for the problem solver's errors. For example:

Menghsha examined the graph of the function $y = 3 \sin \theta$ and determined that the domain of the function was $-1 \leq \theta \leq 1$. Is Menghsha's answer correct? If not, provide the correct answer and explain Menghsha's error.

- Given one method of solving a problem, solve it a second way. For example:

Jillian was asked to find the factors of $P(x) = x^3 - 9x^2 - x + 9$. On her graphing calculator, she graphed the function and determined that the factors for $P(x)$ were $(x - 9)$, $(x + 1)$, and $(x - 1)$. If Jillian was unable to graph $P(x)$, show another method that Jillian could have used to find its factors.

- Given that a Ferris wheel with a radius of 18 m makes a complete revolution in 12 s, draw a diagram of the situation and create a table of values showing the relationship between the height h of a rider above the ground (the lowest point of the Ferris wheel is 1 m above the ground) and the time t to determine the height of a rider after 6 s.

The student demonstrating excellent achievement can:

- Given a problem, solve it for the specific case(s) and then provide a general solution. For example:

Given a 4-sided and a 10-sided polygon, determine the number of diagonals in each. The student can also determine a general statement about the number of diagonals in an n -sided polygon.

- Given that a Ferris wheel with a radius of 18 m makes a complete revolution in 12 s, develop a mathematical model that describes the relationship between the height h of a rider above the lowest point of the Ferris wheel, which is 1 m above the ground, and the time t . The student can provide a full explanation of how his or her model was developed and can suggest alternative ways of developing the model.

¹Italicized comments give an overview of the curriculum statements found in the *Mathematics 30 Course of Studies*.

Polynomial Functions

Given an integral polynomial function of degree 3 or less, students should be able to determine its zeros, its factors, and its graph, and should be able to describe, orally and in writing, the relationship among its zeros, its factors, and its graph.

The student demonstrating acceptable achievement can:

- Given $P(x) = 10x^3 + 51x^2 + 3x - 10$, determine its zeros, its factors, and its graph. The student can also describe the relationship among its zeros, its factors, and its graph.

The student demonstrating excellent achievement can:

- Given $P(x) = ax^3 + bx^2 + 3$ and that the remainders are 7 and 10 when divided by $x - 2$ and $x + 1$ respectively, determine the zeros, the factors, and the graph of $P(x)$.

Trigonometric and Circular Functions

Students should be able to solve a first-degree equation that involves the primary trigonometric functions and describe the relationship between its root(s) and the graph of its corresponding function.

Students should also be able to demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.

The student demonstrating acceptable achievement can:

- Given $y = 2 \sin\left(\theta - \frac{1}{2}\right)$, $0 \leq \theta < 2\pi$, determine its zeros, describe orally and in writing the relationship between its zeros and its corresponding graph, and describe the effect that 2 and $-\frac{1}{2}$ have on the graph of $y = \sin \theta$.
- Given $\frac{\cot \theta}{\tan \theta}$, use fundamental trigonometric identities to simplify the expression and verify this simplification by substituting values for the variable and by comparing their corresponding graphs.

The student demonstrating excellent achievement can:

- Given $2 - 2 \cos^2 \theta = \sin \theta$, $0 \leq \theta < 2\pi$, determine its zeros and describe orally and in writing the relationship between its zeros and the graphs of $y = 2 - 2 \cos^2 \theta$ and $y = \sin \theta$.

Statistics

Given a set of normally distributed data, students should be able to describe and analyze the data using the characteristics of a normal distribution.

The student demonstrating acceptable and the student demonstrating excellent achievement can:

- Given that the results of a test were normally distributed with a mean of 30 and a standard deviation of 5, and given that the passing mark was set at 25, determine the percentage of students who passed the test.

Quadratic Relations

Students should be able to describe orally, in writing, and by modelling, the quadratic relation resulting from the intersection of a plane and a circular conical surface and, from the graph of a quadratic relation, the combination of values for the numerical coefficients of the general quadratic relation that defines each graph and that would result in the degenerate quadratic relations. Given the locus defining a quadratic relation and/or the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, they can identify the quadratic relation described.

The student demonstrating acceptable achievement can:

- Given $2x^2 + 2y^2 + x - 3y - 25 = 0$, identify the quadratic relation described by this equation.
- Given $Ax^2 + Cy^2 + Dx - Ey - 36 = 0$, identify this as a hyperbola when $AC < 0$.
- Given that a quadratic relation is represented by $3x^2 + 4y^2 + 5x + Ey - 36 = 0$, where $B = 0$, describe orally or in writing what happens to the graph of this quadratic relation when 5 is changed to -4 and -36 is changed to -9 .
- Given a quadratic relation that is described as having an eccentricity of 2, identify this as a hyperbola and describe its locus.
- Given that the locus of points such that the sum of the distances between one of the points and two fixed points is constant, identify this locus as an ellipse.
- Given a description of the intersection of a plane and a conical surface, identify the conic section formed.

- Given that the cutting plane approaches the vertex of the conical surface, describe orally, in writing, and by modelling, the effect on the ellipse.
- Given that the eccentricity of the orbit of Halley's comet, which has a period of 76 years, is 0.96, sketch its graph.

The student demonstrating excellent achievement can:

- Describe and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change.
- Given a description of the intersection of a plane and a conical surface, identify orally or in writing the degenerate parabola formed.
- Given the equation of a degenerate quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, describe orally and in writing the quadratic relation formed.
- Given the graph and the eccentricity of a quadratic relation, describe orally and in writing the changes to the graph when the eccentricity changes.
- Given that the fixed point of an ellipse is moving closer to the centre of the ellipse, describe orally and in writing the effect on the eccentricity.
- Given that the eccentricity of any quadratic relation is the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, describe orally and in writing the effect that changing the eccentricity has on the relative positions of the fixed line and the fixed point.

Exponential and Logarithmic Functions

Given an exponential function, students should be able to describe orally and in writing its inverse as the logarithmic function and what this information means in the solution of exponential equations.

The student demonstrating acceptable achievement can:

- Given $f(x) = 4^{2x}$, sketch its graph, describe its domain and range, find the zeros of its corresponding equation, and describe orally and in writing the relationship between the roots of its equation and its graph. The student can write the inverse of $f(x) = 4^{2x}$ in logarithmic form, sketch its graph, describe its domain and range,

determine the zeros of this equation, and describe orally and in writing the relationship between the zeros of its equation and its graph.

The student demonstrating excellent achievement can:

- Given the equation $\log_5(x - 4) + \log_5(x - 2) = 3$, find all the possible values of x , identify the domain, describe orally and in writing the relationship between the roots of this equation and its graph, and describe orally and in writing the reasons why there are values of x that satisfy the equation but are not permissible for the function.

Permutations and Combinations

Students should be able to describe orally and in writing the difference between a permutation and a combination and calculate the number of permutations or combinations of n things taken r at a time.

The student demonstrating acceptable achievement can:

- Given that there are 10 musicians in the finals of a music competition, decide whether permutations or combinations should be used to calculate the number of ways in which first, second, and third prizes can be awarded.
- Given the binomial $(x + 2)^5$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing $x^4(2)$ is obtained, determine the number of terms in the expansion of $(x + 2)^5$, and describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial.

The student demonstrating excellent achievement can:

- Given that five people can sit at a round table, decide whether permutations or combinations should be used to determine the number of different orders in which these five can sit at the table if Jack and Jill must sit next to one another. The student can also justify the method of solution.
- Given the binomial $(3x - 2y)^7$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing $[(3x)^6(2y)]$ is obtained, and determine the number of terms in the expansion of $(3x - 2y)^7$. The student can describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial, how the number of a given term in the expansion of $(3x - 2y)^7$ relates to the exponent of $(2y)$ in that term, and how the coefficients of the

terms that are equidistant from the ends of the expansion of $(3x - 2y)^7$ compare in terms of combinations.

Sequences and Series

Given finite arithmetic and geometric sequences, finite arithmetic series, or finite geometric series, students should be able to describe orally and in writing the differences between sequences and series, describe orally and in writing the differences between finite and infinite, determine the terms of arithmetic and geometric sequences, and determine the sums of arithmetic and geometric series.

The student demonstrating acceptable achievement can:

- Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, and determine the sum of a specified number of terms.
- Given that a series is defined by $\sum_{n=3}^6 (-2)^n$, write the terms of the series, determine whether the series is arithmetic or geometric, and determine the sum of the series.

The student demonstrating excellent achievement can:

- Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, determine the sum of a specified number of terms, and determine the formula for the sum of n terms.
- Given an arithmetic sequence where $t_4 + t_{13} = 99$ and $t_7 = 39$, determine the first term of this sequence.

Examination Specifications

Each Mathematics 30 Diploma Examination is designed to reflect the core content outlined in the *Mathematics 30 Course of Studies*. The examination is limited to those expectations that can be measured by a paper-and-pencil test. The time allotted to write the examination is two and one-half hours. Students may take an additional 0.5 h to complete the examination.

The content for the 1996 Mathematics 30 diploma examinations in the machine-scored section is emphasized as follows:

<i>Content²</i>	<i>Percent Emphasis</i>
Polynomial Functions	11
Trigonometric and Circular Functions	12
Statistics	5
Quadratic Relations	10
Exponential and Logarithmic Functions	11
Permutations and Combinations	10
Sequences and Series	11
<i>Total Multiple Choice and Numerical Response</i>	70
<i>Written Response</i>	30

As published in the 1994-95 Mathematics 30 Diploma Examination Information Bulletin, the written-response questions will assess whether or not students can draw on their mathematical experiences to solve problems and to explain mathematical concepts. Therefore, the written-response questions will not necessarily fall into a particular unit of study but may cross more than one unit or may require students to make the connections between mathematical concepts.

The three mathematical understandings³ of Procedures, Concepts, and Problem Solving are addressed throughout the examination. Each understanding has the following emphasis:

<i>Multiple Choice and Numerical Response</i>	<i>Percent Emphasis</i>
Procedures	21
Concepts	24.5
Problem Solving	24.5
<i>Written Response</i>	
Procedures, Concepts, Problem Solving	30

²Content descriptions have been shortened in this table.

³An explanation of mathematical understandings is given in Appendix G.

Mathematics as Communication

Each examination is built as closely as possible to these specifications.

In keeping with the expectations listed in the *Mathematics 30 Course of Studies*, the 1996 examinations will reflect mathematics as communication. The program of studies includes communication in the problem-solving expectations: “Students will be expected . . . to read the problem thoroughly; identify and clarify key components; restate the problem, using familiar terms . . . ask relevant questions . . . document the solution process . . . and explain the solution in oral or written form . . .” (From *Mathematics 30 Course of Studies*, pp. 6–7).

These expectations are consistent with the recommendations made by the National Council of Teachers of Mathematics:

In grades 9–12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about;
- appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.

Focus: All students need extensive experience listening to, reading about, writing about, speaking about, reflecting on, and demonstrating mathematical ideas.

(From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 140)

The National Council of Teachers of Mathematics describes the evaluation of mathematics as communication in the following manner:

The assessment of students’ ability to communicate mathematics should provide evidence that they can

- express mathematical ideas by speaking, writing, demonstrating, and depicting them visually;
- understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms;
- use mathematical vocabulary, notation, and structure to represent ideas, describe relationships, and model situations.

(From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 214)

Besides writing to communicate results, the accuracy of and logic in students' mathematical statements also reflects mathematics as communication. Hence, students will continue to be expected to demonstrate logical and meaningful communication on the diploma examinations.

Mathematics as Problem Solving

In keeping with the expectations identified in the *Mathematics 30 Course of Studies*, the 1996 examinations will reflect mathematics as problem solving. Problem solving is integrated throughout the content areas in the curriculum. A set of specific problem-solving learner expectations precedes the specific content learner expectations.

The expectations contained in the program of studies are consistent with the recommendations made by the National Council of Teachers of Mathematics:

In grades 9–12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations.

Focus: In grades 9–12, the problem-solving strategies learned in earlier grades should have become increasingly internalized and integrated to form a broad basis for the student's approach to doing mathematics, regardless of the topic at hand. From this perspective, problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics as identified in later standards is both constructed and reinforced.

(From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 137)

Examination Design

The design of the 1996 Mathematics 30 diploma examinations is as follows:

<i>Question Format</i>	<i>Number of Questions</i>	<i>Percent Emphasis</i>
Multiple Choice	40	57
Numerical Response	9	13
Written Response	3	30

See Appendix D for a list of directing words that may be used in the written-response section of the examination.

The 1996 examinations consists of a multiple-choice section, a numerical-response section, and a written-response section.

Multiple Choice

In the multiple-choice section, students are to choose the correct or best possible answer from four alternatives.

Numerical Response

In the numerical-response section, students are to calculate a numerical answer. As well, students are to record their answer on a separate answer sheet, usually correct to the nearest tenth or nearest hundredth. When the answer students are to record is not a decimal value (e.g., the number of people or the degree of a polynomial), students are asked to determine what “the number of people is _____” or “the degree of this polynomial is _____.” If the answer can be a decimal value, then students are asked to record their answer correct to the nearest tenth or nearest hundredth.

Written Response

The written-response section focuses on students’ understanding of the process of solving a problem and encourages students to take risks to arrive at a solution. Students will be rewarded for selecting a problem-solving strategy and for carrying through with the strategy to find a solution. To achieve *excellence*, students must be able to select a strategy, carry it through, and complete the problem. The written-response section of the examination also focuses on students’ understanding of mathematical concepts and allows for the most flexibility in gaining an understanding of students’ communication and problem-solving abilities in mathematics.

Questions in the written-response section ask students to solve, explain their solution, justify their solution, or prove. For a definition of directing words that students may encounter on the diploma examination, see Appendix D.

In scoring the written-response section of the examinations, markers will evaluate students for how well they:

- understand the problem or the mathematical concept
- correctly use the mathematics
- use problem-solving strategies and explain their answer and procedures
- communicate their solutions and mathematical ideas.

Above all, students should be encouraged to try to solve the problem. Even an attempt at a solution could be worth some marks. If students leave the paper blank, markers will not be able to award any marks. For a look at a sample of responses to

written-response questions on the January 1995 examinations, and to see how they were scored, see Appendix H. Please share the information found in Appendix E and F with your students, on preparing for and writing a diploma examination.

Assessment of Communication and Problem Solving

The open-ended question is a way in which to examine mathematics as communication and mathematics as problem solving. An open-ended question allows students to communicate a response by asking them to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

Alberta Education recommends the following two documents, which examine the open-ended question in further detail:

Assessment Alternatives in Mathematics: An overview of assessment techniques that promote learning.

A Question of Thinking: A First Look at Students' Performance on Open-Ended Questions in Mathematics.

Both these documents are available through:

California State Department of Education
Bureau of Publications, Sales Unit
P. O. Box 271
Sacramento, CA 95802-0271

The following document was recently published by the National Council of Teachers of Mathematics. It provides some practical suggestions for the assessment of problem solving and communication on a regular basis:

Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions.

This document is available through:

National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091-1593

Calculator Policy

POLICY: USE OF CALCULATORS ON ALBERTA EDUCATION DIPLOMA EXAMINATIONS

Background

The knowledge, skills, and attitudes relevant to technology and its uses are being incorporated into courses and programs of study wherever appropriate. Students are expected to learn the advantages and limitations of technological developments and their impact upon society. The ability to use technology helps students understand and appreciate the process of technological change, gives added depth to programs, and provides the basis for the development of skills and understanding. These expectations are reflected in the diploma examinations. Since the data provided for writing diploma examinations in mathematics and the sciences do not include information such as logarithms and trigonometric functions, students will need to use scientific calculators for these exams.

Definition

This policy considers a scientific calculator to be a hand-held device designed primarily for mathematical computations. Included in this definition are those scientific calculators having graphing capabilities, built-in formulas, mathematical functions, or other programmable features.

Policy

To ensure compatibility with provincial *Programs of Study* and equity and fairness for all students, Alberta Education expects students to use scientific calculators, as defined above, when they are writing diploma examinations in mathematics and the sciences. Examinations are constructed to ensure that the use of particular models of calculators neither advantages nor disadvantages individual students.

Procedures

1. Teachers must, at the beginning of a course, advise students of the types of calculators that they may use when writing diploma examinations in mathematics and the sciences. Teachers must also advise students of the types of information that can be stored in calculators that are brought into diploma examinations. Calculators that have built-in notes (definitions or explanations in alpha notation) that cannot be cleared are not permitted.
2. Students must clear calculators that are brought into diploma examinations of all information they have stored except for
 - a. programs used for computing values of the formulas on the diploma examination data tear-out pages or in the data booklets,
 - b. programs used for graphing quadratic relations as found in the Mathematics 30/33 Interim Teacher Resource Manual.
3. Students must not bring external devices to support calculators into the exam. Such devices include manuals, printed or electronic cards, printers, memory expansion chips or cards, external keyboards, or any annotations outlining operational procedures for scientific calculators.
4. In preparation for calculator failure, students may bring extra calculators and batteries into the exam room.
5. During exams, supervising teachers must ensure that
 - a. all calculators operate in silent mode,
 - b. students do not share calculators or information contained within them,
 - c. calculator cases are stored on the floor throughout the exam, and
 - d. all examination rules are followed.
6. If you have any questions or comments about the implementation of this policy, please contact the Math/Science Unit, Student Evaluation Branch, at 403-427-0010 or FAX 403-422-4200.

Examination Rules

Appendix B

GRADE 12 DIPLOMA EXAMINATIONS

1. Admittance to the Examination Room

Students must not enter or leave the examination room without the consent of the supervising teacher.

2. Student Identification

Students must present identification that includes their signature and photograph. One of the following documents is acceptable: driver's licence, passport, or student identification card. Students must not write an examination under a false identity or knowingly provide false information on an application form.

3. Identification on Examinations

Students must not write their names or the name of their school anywhere in or on the examination booklet, other than on the back cover.

4. Time

Students must write an examination during the specified time and may not hand in a paper until at least one hour of the examination time has elapsed. Students who arrive more than one hour after an examination has started will not be allowed to write the examination. Students who arrive late but within the first hour of an examination sitting may be allowed to write only at the discretion of the supervising teacher.

5. Discussion

Students must not discuss the examination with the supervising teacher unless the examination booklet is incomplete or illegible. Students must not talk, whisper, or exchange information with one another.

6. Answer Sheets

Students must use an HB pencil to record their answers on the machine-scorable answer sheets.

7. Materials Allowed:

• *English 30, English 33:*

For Part A only, all students may use a print or electronic dictionary (English language or translation), a thesaurus, and an authorized writer's handbook (for example: *The Canadian Writer's Handbook*, Prentice Hall).

• *Français 30:*

For Partie A only, all students may use a print or electronic dictionary (French language or translation), a thesaurus, and a book of verb forms/tenses.

• *Biology 30, Mathematics 30, Physics 30:*

Tear-out data pages are provided in the examination booklets. Students need to use scientific calculators, but must not share them.

• *Chemistry 30, Mathematics 33, Science 30:*

Students must use the separate data booklet provided for each of these examinations. Students need to use scientific calculators, but must not share them.

• *All mathematics and science examinations:*

Students may use calculator programs designed to perform mathematical computations and/or those designed to assist students in graphing.

8. Materials Not Allowed:

• *All Subjects:*

Students must not bring any papers, notes of any kind, books (other than those allowed, see #7) into the examination room. Students must ensure that dictionaries (print and electronic), references, and calculators are completely free of notes or study material.

• *Biology 30, Chemistry 30, Mathematics 30, Mathematics 33, Physics 30, Science 30, Social Studies 30, Social Studies 33:*

Reference materials and dictionaries (translation, English and/or French language) are not allowed.

Appendix C

Guidelines for Significant Digits, Manipulation of Data, and Rounding in the Mathematics and Sciences Diploma Examinations

Significant Digits

1. For all non-logarithmic values, regardless of decimal position, any of the digits 1 to 9 is a significant digit; 0 may be significant. For example:

123 0.123 0.00230 2.30×10^3
all have 3 significant digits
2. Leading zeros are not significant. For example:

0.12 and 0.012 each have two significant digits
3. Trailing zeros to the right of the decimal are significant. For example:

0.123 00 and 20.000 each have five significant digits
4. Zeros to the right of a whole number are considered to be ambiguous. **The Student Evaluation Branch considers all trailing zeros to be significant.** For example:

200 has three significant digits
5. For logarithmic values, such as pH, any digit to the left of the decimal is **not** significant. For example:

a pH of 1.23 has two significant digits
a pH of 7 has no significant digits

Manipulation of Data

1. When adding or subtracting measured quantities, the calculated answer should be rounded to the same degree of precision as that of the least precise number used in the computation **if this is the only operation.** For example:

12.3 (least precise)
0.12
12.34
24.76

The answer should be rounded to 24.8.

2. When multiplying or dividing measured quantities, the calculated answer should be rounded to the same number of significant digits as are contained in the quantity with the fewest number of significant digits **if this is the only operation**. For example:

$$(1.23)(54.321) = 66.81483$$

The answer should be rounded to 66.8.

3. When a series of calculations is performed, each interim value should not be rounded before carrying out the next calculation. The final answer should then be rounded to the same number of significant digits as are contained in the quantity with the fewest number of significant digits. For example:

In determining the value of $(1.23)(4.321)/(3.45 - 3.21)$, three calculations are required:

a. $3.45 - 3.21 = 0.24$

b. $(1.23)(4.321) = 5.31483$

c. $5.31483/0.24 = 22.145125$

[Not $5.31/0.24 = 22.125$]

The value should be rounded to 22.1.

Note: In the example given, steps *a* and *b* yield interim values. These values should not be used in determining the number of significant digits.

Rounding

1. When the first digit to be dropped is less than or equal to 4, the last digit retained should not be changed. For example:

1.2345 rounded to three digits is 1.23

2. When the first digit to be dropped is greater than or equal to 5, the last digit retained should be increased by one. For example:

12.25 rounded to three digits is 12.3

Appendix D

Directing Words

Discuss

The word “discuss” will not be used as a directing word on math and science diploma examinations because it is not used consistently to mean a single activity.

The following words are specific in meaning.

Contrast/Distinguish

Point out the *differences* between two things that have similar or comparable natures.

Compare

Show the character or relative values of two things by pointing out their *similarities* and *differences*.

Conclude

State a logical end based on reasoning and/or evidence.

Criticize

Point out the *merits* and *demerits* of an item or issue.

Define

Provide the essential qualities or meaning of a word or concept. Make distinct and clear by marking out the limits.

Describe

Give a written account or represent the characteristics of something by a figure, model, or picture.

Design/Plan

Construct a plan, i.e., a detailed sequence of actions, for a specific purpose.

Enumerate

Specify one by one or list in concise form and according to some order.

Evaluate

Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages.

Explain

Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail.

How

Show in what manner or way, with what meaning.

Hypothesize

Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.

Identify

Recognize and select as having the characteristics of something.

Illustrate

Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram.

Infer

Form a generalization from sample data; arrive at a conclusion by reasoning from evidence.

Interpret

Tell the meaning of something; present information in a new form that adds meaning to the original data.

Justify/Show How

Show reasons for or give facts that support a position.

Outline

Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps.

Predict

Tell in advance on the basis of empirical evidence and/or logic.

Prove

Establish the truth, validity, or genuineness of something by giving factual evidence or logical reasons.

Relate

Show logical or causal connection between things.

Solve

Give a solution for a problem; i.e., explanation in words and/or numbers.

Summarize

Give a brief account of the main points.

Trace

Give a step-by-step description of the development.

Why

Show the cause, reason, or purpose.

Appendix E

Preparing to Write a Mathematics 30 Diploma Examination—What every student should do before the examinations!

1. Prepare a course review schedule:
 - design your schedule for the two-week period (minimum) before the examination
 - divide the course material into sections and indicate on the schedule the time blocks to be devoted to each section
 - take into account the examination blueprint available from your teacher (*Mathematics 30 Information Bulletin, Diploma Examinations Program*). Note that course units are not equally weighted on the diploma examination
 - take into account units/concepts that you find most difficult; i.e., allocate more time for the review of these
2. Obtain and review examination schedules, rules, and policies:
 - record the time and place of writing
 - note minimum and maximum writing times permitted
 - prepare to remain in the examination room for at least 2.5 h (Kleenex, cough drops, etc.)
 - identify materials allowed for writing each examination, such as pencils, pens, calculators, mathematical instruments, and clear plastic ruler
3. Identify and collect examples of each type of question that will be asked:
 - obtain a copy of the relevant information contained in the *Mathematics 30 Information Bulletin, Diploma Examinations Program* (available from your teacher)
 - review the format of previous diploma examinations (available from your teacher)
 - learn the meanings of key “directing” words such as compare, describe, evaluate, explain, illustrate, interpret, justify, prove, and solve
4. Make summaries and point form outlines:
 - distinguish between major concepts and factual details
 - identify essential skills that can be assessed on paper and pencil tests
 - review lab results and procedures— identify connections between lab reports, class notes, and textbook
 - anticipate examples of connections between concepts and the “real world”
 - prepare a glossary of important subject terminology
 - link each formula or equation with a calculation done on a previous test or assignment
 - identify any restriction on the use of each formula or equation
5. Use memory aids such as:
 - colour coding, underlining, highlighting, jotting key words in margins
 - numbering points to be memorized
 - grouping word and idea associations
 - reading aloud key words, expressing key words in your own words
6. Review the different question formats and the instructions on how to answer these questions.

Appendix F

Suggestions for Students When Writing a Mathematics 30 Diploma Examination—What every student should know when writing examinations!

1. Do not be afraid to answer each question even if you are not sure of the correct solution to the problem. A penalty is NOT given for guessing on the machine-scored section the exam. Partial marks are often awarded for incomplete answers in the written-response section of the exam.
2. If you are stuck on a question, mark the alternatives that you know are incorrect and choose from the ones that are left, using logical guessing strategy. Think of the questions as challenges and cultivate a positive attitude about your ability to answer them.
3. Scan the sets of questions of the examination before answering a particular question. The questions in one set of the examination may jog your memory about a question in another set.
4. When first reading a multiple-choice question, locate and circle key words to help clarify the meaning of the question. Then hide the alternatives and try to formulate an answer of your own. Your answer may be very close to the correct alternative.
5. If a multiple-choice question involves a calculation, do the calculation and select the alternative that is closest to your answer. A multiple-choice calculation is usually short. If you cannot do it in five minutes, your method is either inappropriate or incorrect. Go on.
6. Diagrams on examinations are often labelled with numbers or letters. It may be useful to write in the names of the labelled structures or features that you can identify.
7. When reading graphs, use a clear plastic ruler to more accurately extrapolate or interpolate data.
8. Have a good reason for changing an answer. Do not change an answer on a hunch. Do not waste your time looking for patterns of As, Bs, Cs, or Ds in multiple-choice answers. There are none.
9. You may not have time to write and edit a complete rough copy for each written-response question, but you should prepare an outline of your answer and use it as a guide when writing your good copy.
10. When completing a written-response question, keep in mind the reader of your response. The reader will want to know how well you:
 - understand the problem or the mathematical concept
 - can correctly use the mathematics involved
 - can use problem-solving strategies and explain your answer and procedures
 - can communicate your solutions and mathematical ideas
11. Rewriting a statement of the question is often a good way to begin a written response. Conclude with a summary statement. Be sure you have clearly explained all assumptions and have verified your conclusions.
12. Keep track of the time and pace yourself. Put a check mark by items that you are uncertain about and return to them if there is time at the end of the examination.

Appendix G

Explanation of Mathematical Understandings

Procedures

The assessment of students' knowledge of *mathematical procedures* should provide evidence that they can:

- recognize when a procedure is appropriate;
- give reasons for the steps in a procedure;
- reliably and efficiently execute procedures;
- verify the results of procedures empirically (e.g., using models) or analytically;
- recognize correct and incorrect procedures;
- generate new procedures and extend or modify familiar ones;
- appreciate the nature and role of procedures in mathematics.

It is important that students know how to execute mathematical procedures reliably and efficiently: a knowledge of procedures involves much more than simple execution. Students must know when to apply them, why they work, and how to verify that they have given a correct answer; they also must understand concepts underlying a procedure and the logic that justifies it. Procedural knowledge also involves the ability to differentiate those procedures that work from those that do not, and the ability to modify them or create new ones. Students must be encouraged to appreciate the nature and role of procedures in mathematics; that is, they should appreciate that procedures are created or generated as tools to meet specific needs in an efficient manner and thus can be extended or modified to fit new situations. The assessment of students' procedural knowledge, therefore, should not be limited to an evaluation of their facility in performing procedures: it should emphasize all the aspects of procedural knowledge addressed in this standard.

Concepts

The assessment of students' knowledge and understanding of *mathematical concepts* should provide evidence that they can:

- label, verbalize, and define concepts;
- identify and generate examples and non-examples;
- use models, diagrams, and symbols to represent concepts;
- translate from one mode of representation to another;
- recognize the various meanings and interpretations of concepts;
- identify properties of a given concept and recognize conditions that determine a particular concept;
- compare and contrast concepts.

In addition, assessment should provide evidence of the extent to which students have integrated their knowledge of various concepts.

An understanding of mathematical concepts involves more than mere recall of definitions and recognition of common examples: it encompasses the broad range of abilities identified in this standard. Assessment, too, must address these aspects of conceptual understanding. Assessment tasks should focus on students' abilities to discriminate between the relevant and the irrelevant attributes of a concept in selecting examples and non-examples, to represent concepts in various ways, and to recognize students' various meanings. Tasks that ask students to apply information about a given concept in novel situations provide strong evidence of students' knowledge and understanding of that concept. Problems designed to elicit information about students' misconceptions can provide information useful in planning or modifying instruction.

Problem Solving

The assessment of students' ability to use mathematics in *solving problems* should provide evidence that they can:

- formulate problems;
- apply a variety of strategies to solve problems;
- solve problems;
- verify and interpret results;
- generalize solutions.

Students' ability to solve problems develops over time as a result of extended instruction, opportunities to solve many kinds of problems, and encounters with real-world situations. Students' progress should be assessed systematically, deliberately, and continually to effectively influence their confidence and ability to solve problems in various contexts. Assessments should determine students' ability to perform all aspects of problem solving. Evidence about their ability to ask questions, use given information, and make conjectures is essential to determine if they can formulate problems. Assessments also should yield evidence of students' use of strategies and problem-solving techniques and of their ability to verify and interpret results. Finally, because the power of mathematics is derived, in part, from its generalizability, this aspect of problem solving should be assessed as well.

From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 209, p. 223, p. 228.

Appendix H

Looking at Sample Responses

This section of the bulletin will provide some samples of responses to two of the four written-response questions from the January 1995 diploma examination.

2. In the event that the school is closed for unforeseen reasons, the principal would like to use a “telephone fan-out system” to contact all staff members.

In this “telephone fan-out system,” the principal telephones n people and then each one of these people telephones n more people. This pattern continues until all staff members are telephoned.

- Draw and label a diagram that models such a “telephone fan-out system.”
- An equation can be determined to describe the total number of staff members phoned. From your model, determine an equation that connects the total number of people phoned with the value of the variable n . Introduce and define any other variables, as required, to complete your equation.
- What assumption is necessary to construct a diagram that models such a “telephone fan-out system”?
- The principal asks you to design a “telephone fan-out system” so that all staff are telephoned in the least amount of time. What factors will you have to consider so that the system you design will take the least amount of time?

Question 2

Diagram

- | | |
|---|--|
| 2 | The student draws and labels a diagram that correctly models such a “telephone fan-out system.” |
| 1 | The student draws a diagram that models a “telephone fan-out system” (the next level meaning more people contacted) but does not clearly communicate an understanding of this problem. |
| 0 | No diagram is provided. |

Equation

- 3** The student's equation reflects the model (or a more general model) of such a "telephone fan-out system" and represents the total number of people phoned, with the variables clearly defined.
- 2** The student's equation reflects the model (or a more general model) of such a "telephone fan-out system" and represents the total number of people phoned; however, the variables are not clearly defined.
- 1** The student attempts to write an equation that represents the total number of people phoned; however, the variables are not clearly defined, nor is the equation a reflection of their model
OR
The student's equation accurately reflects some part of the model and there is an attempt to define the variables; however, the equation does not represent the total number of people phoned.
- 0** The student writes an equation that is not related to the problem.

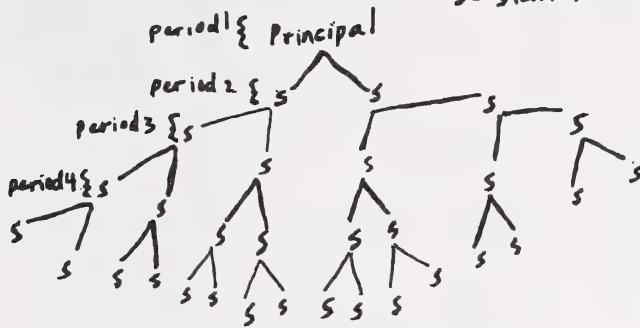
Assumption and Factors

- 2** The student correctly identifies more than two assumption/factors to be considered in designing the "telephone fan-out system."
- 1** The student correctly identifies one or two assumption/factors to be considered in designing the "telephone fan-out system."
- 0** No assumption or factor is identified or the assumption and factors identified are not relevant to the problems.
-

Sample 1

• LABEL DIAGRAM. :

number of staff members = 30
 s = staff member.



$$2(n_1 + n_2 + n_3 + n_4) = 30$$

where:
 n_1 = the number of callers in period 1.
 n_2 = the number of callers in period 2.
 n_3 = the number of callers in period 3.
 n_4 = the number of callers in period 4.

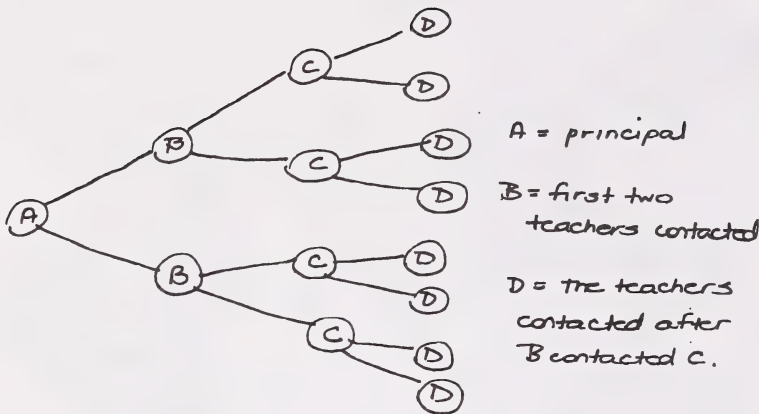
• The assumptions that ^{are} necessary to construct a diagram that models such a "telephone fan-out system" are that each staff member assigned to phone other staff members will phone, and that all staff members have a telephone.

This response would receive a total score of 7. The response would receive 2 marks for the diagram because the diagram correctly models a "telephone fan-out system," and is labelled. The equation would receive a mark of 3 because it reflects the model of the diagram, represents the total number of people phoned in the diagram, and all of the variables are clearly defined. Finally, the response would receive 2 marks for the assumption and factors.

- - the number of staff who have a telephone.
- the number of staff who are at home.
- the number of staff who have an answering machine.
- the length of the message.
- the type of phone (eg. touch tone)

Sample 2

Diagram :



Equation :

number of teachers phoned = 2^{n-1}

$n \rightarrow$ number of letters in the set. Each letter contacts two more letters.

For example, if A phoned B, then C phoned D, the number of people contacted would be

$2^{4-1} = 8$ people contacted.

2^{n-1} = people contacted

$n \rightarrow$ number of letters in the set used including the first letter (principal).

The assumption necessary to construct the diagram that models this system is that, after each step in the fan out system, the number of people telephoned is doubled.

Factors :

So that the system can proceed quickly, it is imperative that each teacher is only assigned only a certain two other members of the staff to phone, so that no staff member is phoned twice, or no staff member has been forgotten.

Also, The staff members must be contacted by the principal before school begins, so that no member of the staff is left uninformed.

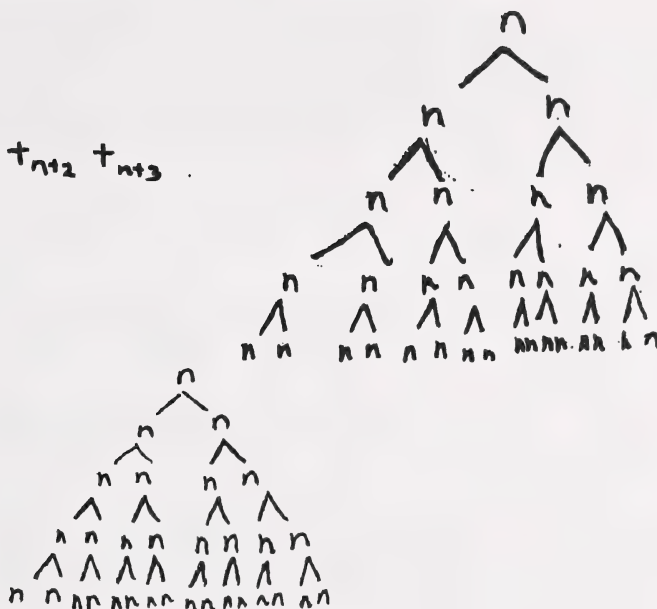
This response would receive a total score of 5. The diagram would receive 2 marks because it models a "telephone fan-out system" and is correctly labelled. The equation would receive 1 mark. The equation reflects the number of people contacted in a "round" of phone calls rather than the total number of people phoned in this model. The response would also receive 2 marks for the assumption and factors.

Sample 3

n

$n + n$

$n + n + n + n + n + n + n + n$



This response would receive a total score of 3. The diagram would receive 1 mark because it does model a "telephone fan-out system," but it is not labelled. The equation would receive a score of 0. An equation is written but it does not reflect any part of the problem. The assumption and factors would receive a score of 2.

$a = n$
 $r = 2$

$t_n = ?$

$$t_n = n2^n - 1 = n2^x - 1$$

The assumption that everyone is available for the call, and that these people will also call 2 other people.

Some factors are if everyone will take the call; if everyone will make the call; and if they do it immediately.

3. Jennifer was given the following problem to solve.

Eight students volunteer to help out at the school dance. Two volunteers are required to check coats, two different volunteers are required to sell pop, and the remaining volunteers are required to clean the gym. How many different ways can the students be selected for the various jobs?

Jennifer considered two models that correctly represent this problem:

Model A	Model B
${}_8C_2 \times {}_6C_2 \times {}_4C_4$	$\frac{8!}{2!2!4!}$

Describe how each model represents the problem.

Question 3

- | | |
|---|---|
| 5 | The student clearly communicates a complete and correct description of how both models represent the problem; all of the elements are acknowledged, but a minor flaw may be present. |
| 4 | The student clearly communicates a correct description of how both models represent the problem; the explanation contains a minor flaw that does not reflect a misunderstanding of the problem. |
| 3 | The student clearly communicates a complete and correct description of how one of the models represents the problem
OR
The student clearly communicates a correct description, which may contain a minor flaw, of how one of the models represents the problem AND begins a correct explanation, which goes beyond simply copying the data given in the problem, of how the other model represents the problem. |
| 2 | The student clearly communicates a correct description of one of the models but the explanation contains a minor flaw, which does not reflect a misunderstanding of the problem
OR
The student begins a correct description of both models, which goes beyond simply copying the data given in the problem. |
| 1 | The student begins a correct description of how one of the models represents the problem, which goes |

beyond simply copying the data given in the problem.

- 0 The student simply copies the data given in the problem, or the student's response is not correct or is not related to the given problem.
-

Sample 1

Model A

At the very beginning, there are 8 jobs and people available, and 2 people are required to check coats. Therefore 2 are chosen from the 8 ($8C2$). These leaves 6 workers and 6 jobs left over. Out of the six, 2 more are required to sell pop. Therefore 2 more are chosen from the 6 ($6C2$). Now there are 4 workers remaining and 4 jobs. 4 people are required to clean the gym so 4 are chosen from the 4 ($4C4$). The values are all multiplied together because 2 are required to check coats, 2 are required to sell pop and 4 are required to clean. When there is an and between the values, you multiply them.

Model B

In this model, because there are 8 jobs and 8 people the people can be matched with a job $8!$ ways. However, there are repetitions. There are 2 people to check coats so you divide by $2!$. There are 2 more who are selling pop, so again you divide by $2!$. Finally because 4 people have the identical job of cleaning the gym, you divide by $4!$.

This response would receive a score of 5 because all elements involved in the problem are addressed, including the description of identical jobs and of the multiplication of the choices in model A.

Sample 2

Model A

There is a total of 8 students to begin with & out of the 8, 2 are needed to check coats thus ${}^8C_2$. 6 students are now left & 2 different students are needed to sell pop, ${}^6C_2$. The remaining 4 will all be used to clean the gym so ${}^4C_4$. ${}^8C_2, {}^6C_2, {}^4C_4$, are multiplied because these all happen together there is no choice.

Model B

There are $8!$ ways to arrange the students but these are indistinguishable. There are 2 needed for coats, 2 for pops & 4 for gym cleanup. To make these distinguishable arrangements $2!2!4!$ must be divided by $8!$

This response would receive a score of 4 because the description does describe how both models model the problem. The response though does not describe that the jobs are indistinguishable for model B, and this does not reflect a misunderstanding of the problem.

Sample 3

Model A would be more appropriate because it relates completely to the problem. In model A, it begins with $8C_2$ which means that out of 8 volunteers, 2 are to be chosen to check coats since, there are only 6 left, Jennifer has to choose 2 more people out of 6 people left which is $6C_2$, then there are 4 more students left and this remaining people are to be chosen clear the gym. But she screwed up on the last step. She does not have to choose out of those 4 students left, since it says in the instruction that she don't have to choose out of those remaining people. Model B, is confusing

$$8C_2 \times 4C_2 \times 1 = 420$$

This response would receive a score of 1. There is the beginning of a correct description of how one of the models represents the problem. The response goes beyond copying the data from the problem when it addresses descending order.

Sample 4

Model A.

$$8C_2 \times 6C_2 \times 4C_1$$

2 students out of a possible 8 students are chosen.
6 students remains, 2 of these are chosen.
4 students are left, and are all given a job.
They are multiplied together because it is not a choice of one or the other being chosen.

Model B

8! There is 8! because 8 students are available, after 1 is chosen, there is 7 left, and 6 left after another is chosen.

They are divided by the number of jobs available at each position. There are factorial because if there are 2 jobs, after one is filled, there is still one available.

This response would receive a score of 2. There is a beginning to a correct description of how both models can represent the problem.

Appendix I

Mathematics 30 Curriculum Standards

The Curriculum Standards provided in this appendix are intended to clarify the *Mathematics 30 Course of Studies* statements. Included are examples of questions that students must be able to do to demonstrate *acceptable* or *excellent* achievement. For a definition of *acceptable* and *excellent* achievement, see pages 2 and 3 of this Bulletin.

Problem Solving

Students in Mathematics 30 can participate in and contribute toward the problem-solving process for problems within the seven content strands.¹

Polynomial Functions

Given any integral polynomial function of degree 3 or less, students can determine its zeros, its factors, and its graph, and can describe, in writing, the relationship among its zeros, its factors, and its graphs.

The student demonstrating acceptable achievement can:

- recognize and give examples of polynomial functions of different degrees;
- generate the graph of any integral polynomial function with the use of graphing calculators or graphing utility packages;
- use the Remainder Theorem to evaluate a third-degree integral polynomial function for rational values of the variable, and to understand how this can be used to find factors of the polynomial function;
- factor and find the zeros for an integral polynomial function in standard form, degree 3 or less, in which all zeros are rational;
- find approximations for all the real zeros of integral polynomial functions, using graphing calculators or computers;
- derive an equation of an integral third-degree polynomial function, given its rational zeros;
- recognize the general shape of graphs of integral polynomial functions of degree 4 or less where the multiplicity of zeros is one, two, or three;
- identify the potential rational zeros of an integral polynomial function;
- determine the minimum degree of a polynomial function by using the multiplicities of its zeros;
- participate in and contribute toward the problem-solving process for problems that can be represented by polynomial functions studied in Mathematics 30.

¹Italicized comments give an overview of the curriculum statements to be found in the *Mathematics 30 Course of Studies*.

The student demonstrating excellent achievement can also:

- use the Remainder Theorem when either the factor or the original polynomial contains unknown coefficients;
- explain the relationships between the graphs of different polynomial functions and their zeros;
- derive an equation for an integral polynomial function, given its zeros and any other information that will uniquely define it;
- use the Remainder Theorem to evaluate integral polynomial function beyond the third degree for rational values of the variable, and understand how this can be used to find factors of the polynomial function;
- recognize the general shape of graphs of integral polynomial functions of degree n where the multiplicity of zeros is greater than two;
- complete the solution to problems that can be represented by polynomial functions studied in Mathematics 30.

Trigonometric and Circular Functions

Students should be able to solve a first-degree equation that involves the primary trigonometric functions and describe the relationship between its root(s) and the graph of its corresponding function.

Students can also demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.

The student demonstrating acceptable achievement can:

- convert angle measurements between degree and radian measure;
- given any two of the following measurements—the radian measure of the central angle, the radius, or the length of an arc—determine the unknown measurement;
- verify² the fundamental trigonometric identities;
- solve first-degree trigonometric equations on the domain $0 \leq \theta < 2\pi$ in radians and $0^\circ \leq \theta < 360^\circ$;
- simplify and evaluate simple trigonometric expressions involving the fundamental trigonometric identities;
- generate the graph of trigonometric functions with the use of graphing calculators or graphing utility packages;
- explain the effect of each parameter a , b , c , and d on the graph of the $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$ functions;
- state the domain and range of $y = \sin \theta$, $y = \cos \theta$, and $y = \tan \theta$;

²For a definition of **verify**, see the *Mathematics 30 Course of Studies*, p. 6.

- describe, orally and in writing, the relationship between the root(s) of a first-degree trigonometric equation and the graph of its corresponding function;
- participate in and contribute toward the problem-solving process for problems that can be represented by trigonometric functions studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- prove³ trigonometric identities;
- explain, orally and in writing, the combined effects of the parameters a , b , c , and d in the trigonometric functions $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$, on the functions' domain and range;
- solve first- and second-degree trigonometric equations including double and half angles on the domain $0 \leq \theta < 2\pi$ and $0^\circ \leq \theta < 360^\circ$;
- describe, orally and in writing, the relationship between the root(s) of a trigonometric equation and the graph of its corresponding function;
- complete the solution to problems that can be represented by trigonometric functions studied in Mathematics 30.

Statistics

Students can describe and analyze data by using the characteristics of a normal distribution.

The student demonstrating acceptable achievement can:

- describe the characteristics of normally distributed data;
- interpret the mean and standard deviation of a set of normally distributed data;
- apply the standard normal curve and the z-scores of data that are normally distributed;
- participate in and contribute toward the problem-solving process for problems that require the analysis of statistics studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- complete the solution to problems that require the analysis of statistics studied in Mathematics 30.

Quadratic Relations

Students can describe the conditions that generate quadratic relations.

The student demonstrating acceptable achievement can:

- describe orally, in writing, and by modelling, the intersection of a plane and a conical surface that would result in a hyperbola, an ellipse, a parabola, and a circle;

³For a definition of **prove**, see the *Mathematics 30 Course of Studies*, p. 6.

- describe orally, in writing, and by modelling, and identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate ellipse and hyperbola;
- recognize each quadratic relation when given the locus definition;
- describe, orally and in writing, the quadratic relation when given the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed horizontal or vertical line;
- generate the graph of quadratic relations with the use of graphing calculators or graphing utility packages;
- describe, orally and in writing, and identify the quadratic relation defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$;
- describe, orally and in writing, and identify the quadratic relation formed when given the value of the eccentricity;
- describe, orally and in writing, and identify the eccentricity when given the quadratic relation;
- describe, orally and in writing, and identify the quadratic relation formed when given the locus definition;
- identify and graph the quadratic relation when given a point on the quadratic relation, a fixed point, and the eccentricity;
- calculate the eccentricity when given a fixed horizontal or vertical line, a fixed point, and a point on the quadratic relation;
- identify and graph the quadratic relation when given the eccentricity, a fixed point, and a fixed horizontal or vertical line;
- describe, orally and in writing, and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when TWO of the numerical coefficients change;
- participate in and contribute toward the problem-solving process for problems that require the analysis of quadratic relations studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- describe, orally and in writing, the combination of values for the numerical coefficients of the general quadratic relation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, that would result in the degenerate conics;
- use the locus definition to verify the equation of each conic section;
- describe, orally and in writing, and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change;
- describe, orally and in writing, and identify the changes in the graph of a quadratic relation when the eccentricity changes;
- describe orally, in writing, and by modelling, and identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate parabola;

- complete the solution to problems that require the analysis of quadratic relations studied in Mathematics 30.

Exponential and Logarithmic Functions

Students can describe the relationship between exponential and logarithmic functions.

The student demonstrating acceptable achievement can:

- generate the graph of exponential and logarithmic functions with the use of graphing calculators or graphing utility packages;
- recognize and sketch the graphs of exponential and logarithmic functions and recognize their inverse relationship;
- convert functions from exponential form to logarithmic form and vice versa;
- apply the laws and properties of logarithms to evaluate logarithmic expressions;
- solve and verify simple exponential and logarithmic equations;
- state the domain and range of the exponential and logarithmic functions;
- use the graphs of the exponential and logarithmic functions to estimate the value of one of the variables, given the other variable;
- participate in and contribute toward the problem-solving process for problems that can be represented by logarithmic or exponential functions studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- solve and verify exponential and logarithmic equations;
- complete the solution to problems that can be represented by logarithmic or exponential functions studied in Mathematics 30.

Permutations and Combinations

All students can describe the difference between a permutation and a combination, calculate the number of permutations or combinations of n things taken r at a time, and apply these to the expansion of binomials.

The student demonstrating acceptable achievement can:

- calculate the number of linear, circle, and ring permutations and permutations with repetitions of n things taken r at a time;
- calculate the number of combinations of n things taken r at a time;
- expand binomials of the form $(x + a)^n$, $n \in \mathbb{W}$ using the Binomial Theorem;
- describe, orally and in writing, the difference between a permutation and a combination;
- participate in and contribute toward the problem-solving process for problems involving permutations and/or combinations, including probability problems studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- expand binomials of the form $(x + by)^n$, $n \in \mathbb{W}$ using the binomial theorem, and determine specific terms of this expansion;
- explain the reason that these are different numbers of permutations when a given number of objects are arranged in a line, a circle, or a ring, or when some of the objects are repeated or identical;
- complete the solution to problems involving permutations and/or combinations, including probability problems studied in Mathematics 30.

Sequences and Series

Students can describe the differences between sequences and series with an emphasis on arithmetic and geometric sequences, can determine the terms of arithmetic and geometric sequences, and can determine the sums of arithmetic and geometric series.

The student demonstrating acceptable achievement can:

- write the specific terms of a sequence, given its defining function;
- expand a series given in sigma notation;
- describe, orally and in writing, the difference between sequences and series, arithmetic or geometric, infinite and finite;
- apply the general term “formula” to arithmetic and geometric sequences;
- apply the sum formula for arithmetic and geometric series;
- participate in and contribute toward the problem-solving process for problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30.

The student demonstrating excellent achievement can also:

- solve problems using the general term and/or sum formulas in which there are two unknowns;
- write the specific terms of a sequence when given its recursive definition;
- determine the functions describing any sequence that has a recognizable pattern;
- complete the solution to problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30.

Mathematics 30 Information Bulletin

